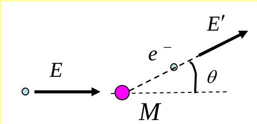
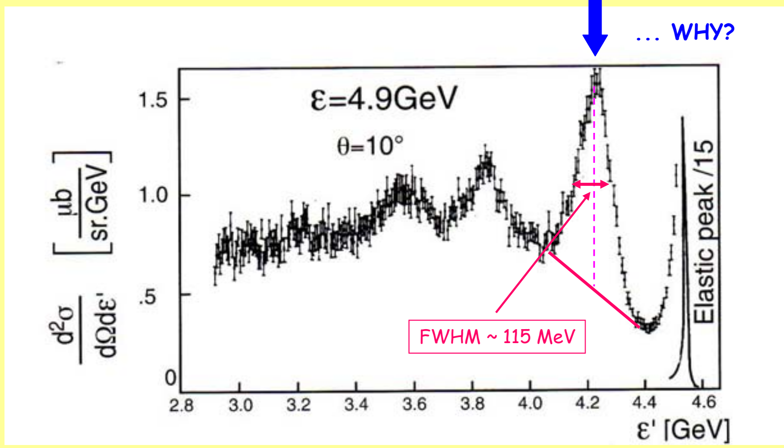
16.451 Lecture 10: Inelastic Scattering from the Proton *Oct. 11, 2005* 1



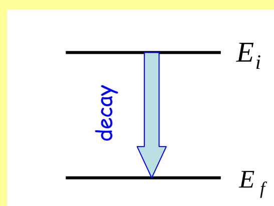
Electron scattering from the proton shows inelastic peaks corresponding to excited states that are very broad in energy:

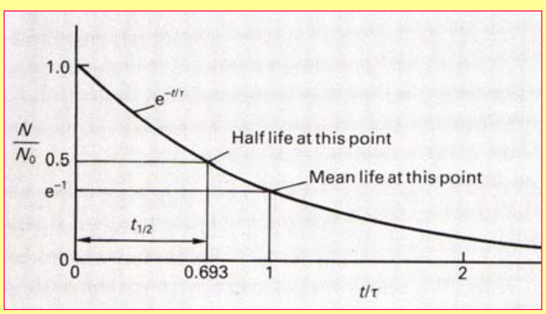
... **WHY?**



Lifetime and Lineshape: *(F&H sec. 5.7)* 2

- Suppose we have a quantum system in unstable state E_i
  → it will decay with some characteristic lifetime τ to the final state E_f
- The transition rate λ_{if} is something we can calculate in principle from Fermi's Golden Rule *(lecture 6!)* if we know the interaction responsible for the decay process....
- With a population N_i in state E_i at time t, we have:





$$dN_i = -N_i\,\lambda_{if}\,dt$$

$$\Rightarrow N_i(t) = N_o\,e^{-\lambda_{if} t}$$

$$= N_o\,e^{-t/\tau}$$

$$t_{1/2} = \tau \ln 2$$

Radioactive Decay Law

continued... 3

The unstable state E_i must be described by a time dependent wave function, such that:

$$\frac{|\psi_i(\vec{r},t)|^2}{|\psi_i(\vec{r},0)|^2} = e^{-t/\tau} = e^{-\Gamma t/\hbar}$$

$$\psi_i(\vec{r},t) = \psi_i(\vec{r},0)\, e^{-iE_i t/\hbar}\, e^{-t/2\tau} = \psi_i(\vec{r},0)\ \exp\left(\frac{it}{\hbar}(E_i - \tfrac{i\Gamma}{2})\right)$$

In order for the decay rate to be correctly described,
the unstable state has to have an energy width $\Gamma = \hbar/\tau$!!!

(or really $i\Gamma/2$; it has to be imaginary for the decay rate to be real!)

This implies we will observe a distribution of energies E, centered on the mean value E_i.

derivation of the line shape function: 4

Let the wave function for the initial state be represented as:

$$\Psi(\vec{r},t) = \psi(\vec{r})\, e^{-iE_i t/\hbar}\ e^{-\Gamma t/2}$$

and since there is an implicit spread of energies of order Γ, we can also write the wave function as a superposition of states of energy E with amplitude a(E):

$$\Psi(\vec{r},t) = \psi(\vec{r}) \int a(E)\, e^{-iEt/\hbar}\ dE$$

compare these expressions and rearrange to find:

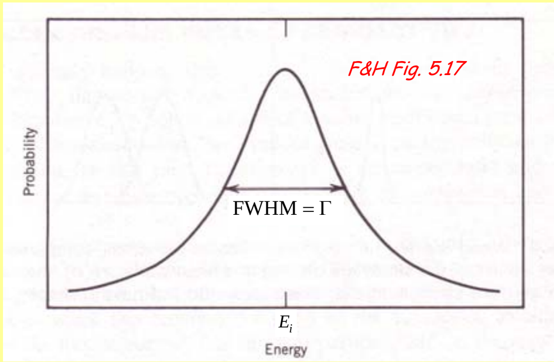
$$e^{-\Gamma t/2} = \int a(E)\, e^{-i(E-E_i)t/\hbar}\ dE$$

this is a Fourier transform relationship! Invert this to find a(E) and **square it** to obtain the probability of finding the initial state with energy E instead of E_i:

$$|a(E)|^2 = \frac{1}{4\pi^2}\ \frac{1}{(E-E_i)^2 + \Gamma^2/4}$$

Lineshape function: 5

The resulting "line shape" for an unstable state is often referred to as a Lorentzian or Breit-Wigner distribution:



shape function:

$$\frac{1}{1+x^2} = \frac{\Gamma^2/4}{(E-E_i)^2+\Gamma^2/4}$$

N.B. $\displaystyle\int_{-\infty}^{\infty}\frac{dx}{1+x^2} = \pi$

(also known as a resonance curve – a short lived state is often called a "resonance")

Application to the proton inelastic scattering data:

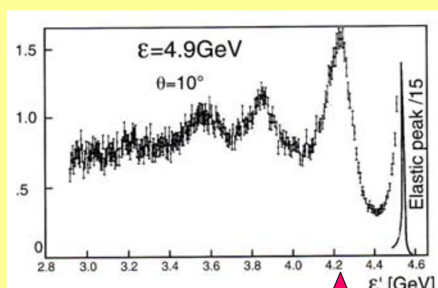
FWHM = Γ = 115 MeV $\rightarrow$ lifetime $\tau = \hbar/\Gamma$ = 5.7×10^{-24} seconds!

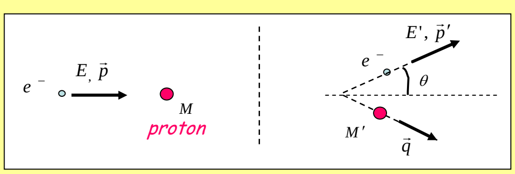
(shorter lifetime implies a larger energy width & vice-versa)

Kinematic analysis: 6

Use kinematics from last class, with E = p, and $\Delta E = (M' - M)$:

$$E' = \frac{E - \Delta E - \frac{\Delta E^2}{2M}}{1 + \frac{E}{M}(1-\cos\theta)}$$





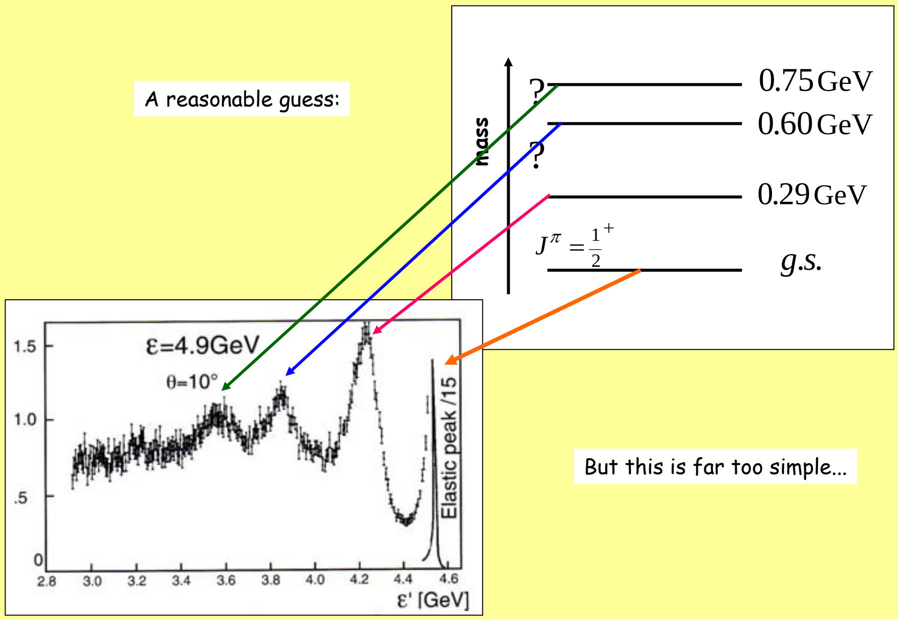
Cross section for 10° scattering -- first inelastic peak is at 4.23 GeV

Result: first excited state is at ΔE = 0.29 GeV, or M' = 1.23 GeV (M = 0.938 GeV)

(the next two have masses M' = 1.52 and 1.69 GeV ...)

What does the energy spectrum of the proton look like? 7

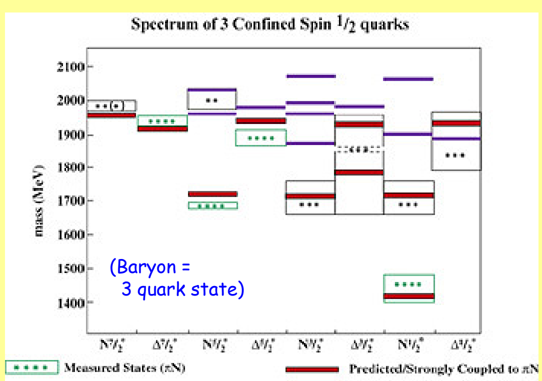
A reasonable guess:



But this is far too simple...

Excited states of the proton and its relatives: 8

The "baryon resonance" energy spectrum is very complex – because the states are all short-lived, the spectrum consists of many overlapping broad states. Careful spectroscopic studies where the decay products are observed and identified are required to sort out the different states according to their angular momentum and parity values....



Only **half** of the predicted states have yet been observed!
http://www.jlab.org/highlights/nuclear/Nuclear.html

First state: the $\Delta(1232)$ *-- there are actually 4 of them!!!* 9

From the Particle Data Group website: http://pdg.lbl.gov -- TRY IT!!

Δ BARYONS
$(S = 0, I = 3/2)$

$\Delta^{++} = uuu, \quad \Delta^{+} = uud, \quad \Delta^{0} = udd, \quad \Delta^{-} = ddd$

$\Delta(1232)$ P_{33} $\qquad I(J^P) = \frac{3}{2}(\frac{3}{2}^+)$

Breit-Wigner mass (mixed charges) = 1230 to 1234 ($\approx$ 1232) MeV

Breit-Wigner full width (mixed charges) = 115 to 125 ($\approx$ 120) MeV

$p_{beam} = 0.30$ GeV/c $\qquad 4\pi\lambda\!\!\!^{-2} = 94.8$ mb

Re(pole position) = 1209 to 1211 ($\approx$ 1210) MeV

-2Im(pole position) = 98 to 102 ($\approx$ 100) MeV

$\Delta(1232)$ DECAY MODES	Fraction (Γ_i/Γ)	p (MeV/c)
$N\pi$	>99 %	227
$N\gamma$	0.52–0.60 %	259
$N\gamma$, helicity=1/2	0.11–0.13 %	259
$N\gamma$, helicity=3/2	0.41–0.47 %	259

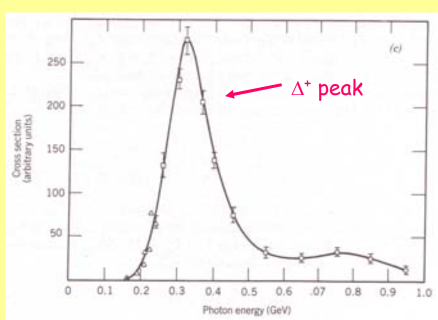
Other evidence: $\Delta(1232)$ 10

- Primary decay mode is $\Delta^+ \rightarrow p + \pi^o$
- the pion, π^o, is the lightest member of the "meson" family, consisting of quark-antiquark pairs. m_π = 140 MeV (compared to the proton, 938 MeV, or the Δ, 1232 MeV)
- the cross-section for photon absorption by the proton, i.e. $\gamma + p \rightarrow X$, peaks at a photon energy that excites the Δ resonance ($E\gamma$ = 340 MeV)
- confirmation is obtained by detecting the proton and pion in the final state and deducing that they have just the right energy to be decay products of a Δ

$$\gamma + p \rightarrow (\Delta) \rightarrow \pi^0 + p$$

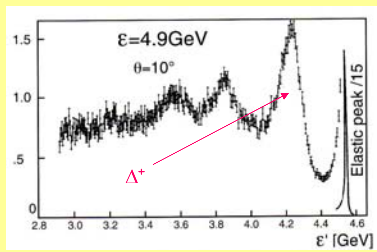
kinematic condition:

p and π^o are emitted back-to-back in the rest frame of the Δ!



Summary: Inelastic scattering from the proton

- a plot of the cross section for inelastic electron scattering (and other processes) shows broad peak structures corresponding to excited states of the proton
- kinematics allows us to determine the mass of the excited state from the scattered electron energy
- peaks are broad because the states are short-lived: FWHM $\Gamma = \hbar/\tau$
- example: Δ^+ "resonance" at 1232 MeV is 294 MeV above the mass of the proton, has a width of 115 MeV, and decays after ~ 6 x 10^{-24} seconds into a proton and a neutral pion.

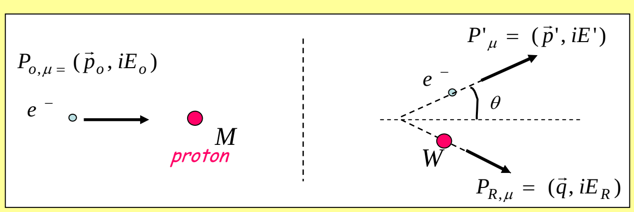


Last topic: Deep inelastic scattering and evidence for quarks

Ref.: D.H. Perkins, Intro to High Energy Physics -- handout

Basic idea goes back to the behavior of form factors:

- $F(q^2) = 1$ (and independent of q^2) for scattering from a pointlike object (lecture 6)
- We are dealing with large momentum transfer, so use the 4-vector description



$$Q = (P_o - P') = \left(\vec{p}_o - \vec{p}\,',\ i\,(E_o - E')\right) \equiv \left(\vec{q},\ i\,\nu\right)$$

Note new definitions: for consistency with high energy textbooks, the symbol W represents the mass of the recoiling object, and ν is the energy transferred by the electron. (careful: $\nu \neq E_R$ because of the mass terms...)

Kinematic analysis: 13

Four momentum transfer:

$$Q = (P_o - P') = \left(\vec{p}_o - \vec{p}',\ i\,(E_o - E')\right) \equiv \left(\vec{q},\ i\,\nu\right)$$

Total energy conservation:

$$E_o + M = E' + E_R \quad \Rightarrow \quad E_R = \nu + M$$

Einstein mass-energy relation for the recoil particle:

$$E_R^2 = W^2 + q^2 \qquad = \nu^2 + 2M\nu + M^2$$

$$Q^2 = q^2 - \nu^2 = 2M\nu + M^2 - W^2$$

mass of recoil

For **elastic** scattering:

$$M = W \quad \Rightarrow \quad Q^2 = 2M\nu$$

$$or \quad x \equiv Q^2 / 2M\nu = 1$$

continued... 14

$$Q^2 = 2M\nu + M^2 - W^2$$

(4-momentum transfer squared)

- For **elastic** scattering:

$$M = W \quad \Rightarrow \quad Q^2 = 2M\nu$$

$$or \quad x \equiv Q^2 / 2M\nu = 1$$

- For **inelastic** scattering:

$$W > M \quad \Rightarrow \quad x < 1$$

<u>Conclusions so far:</u>

- The value of x gives a measure of the inelasticity of the reaction.
- The smaller x is, the larger the excitation energy imparted to the recoiling proton
- x and Q^2 are **independent** variables, and **0 <= x <= 1**

Generalization of the scattering formalism: 15

cross-section:

$$\frac{d^2\sigma}{dQ^2 d\nu} = \frac{4\pi\alpha^2}{Q^4}\frac{E'}{\nu E}\left[\boxed{F_2(Q^2,\nu)}\cos^2(\theta/2) + \frac{2\nu}{M}\boxed{F_1(Q^2,\nu)}\sin^2(\theta/2)\right]$$

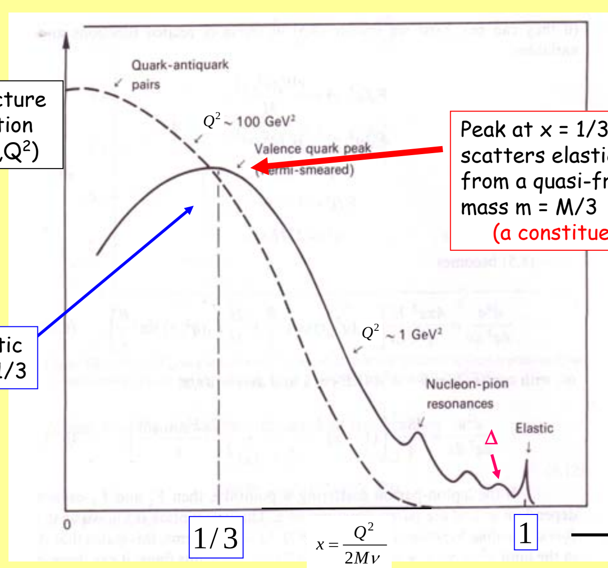
New form factors F_1 and F_2 are called "structure functions" – they depend on both the 4-momentum transfer and the energy transfer.

kinematic factor to classify inelasticity:

$$x = \frac{Q^2}{2M\nu} \ , \quad 0 \le x \le 1$$

Illustration: range of x in electron – proton scattering 16

structure function $F_2(x,Q^2)$



Peak at x = 1/3 if the beam scatters elastically with x = 1 from a quasi-free object of mass m = M/3

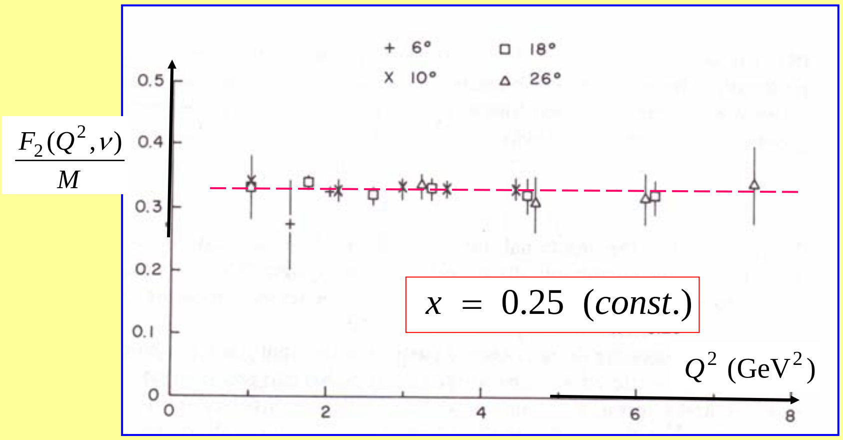
(a constituent quark)

"Deep inelastic region" x = 1/3

$1/3$ $\quad x = \frac{Q^2}{2M\nu} \quad$ $\boxed{1}$ $\longrightarrow x$

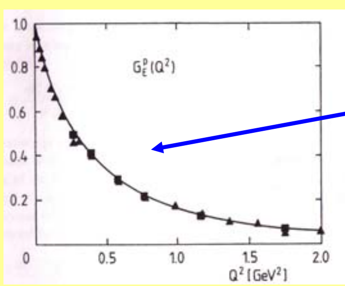
increasing energy transfer ...

Evidence of point-like quarks comes from "scaling" of the structure functions 17

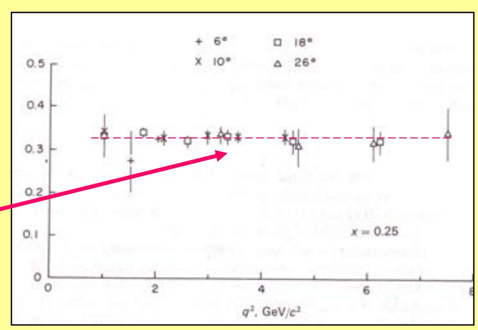


Idea: "structureless" scattering object has a constant form factor or structure function. The proton structure functions are essentially independent of Q^2 in the deep inelastic regime, indicating scattering from pointlike constituents with mass approx 1/3 the proton mass $\rightarrow$ u and d quarks!

Contrast: elastic and inelastic form factors! 18



proton electric form factor for elastic scattering, $x = 1$, falls off rapidly with increasing Q^2



proton "F_2" structure function, deep inelastic regime, $x = 0.25$. independent of Q^2